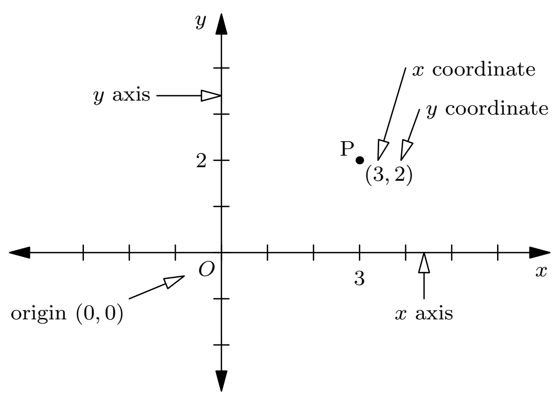
6 Graphs of Straight Lines

6.1 Coordinates *Cartesian*

Every point in the plane requires **2 numbers** *(coordinates)* to fix or describe its position uniquely. Coordinates $(3, 2)$ of the point P in the system (or framework) shown are called *Cartesian coordinates* after the Frenchman, René Descartes, who introduced the system in 1637).



The x value *(x-coordinate)* is listed first.

The y value *(y-coordinate)* is listed second.

Any two letters can be used to specify the *axes* (the x-axis and y-axis in the diagram). The two perpendicular axes are in fact two number lines. Their point of intersection is called the *origin* with coordinates $(0, 0)$.

> *Exercises 6.1:*
> Plot (i.e. mark in the coordinates of) a few points of your own choosing (e.g. $(-2, 1), (1, -3), (-2, -1\frac{1}{2}), (0, 1)$) as an exercise.

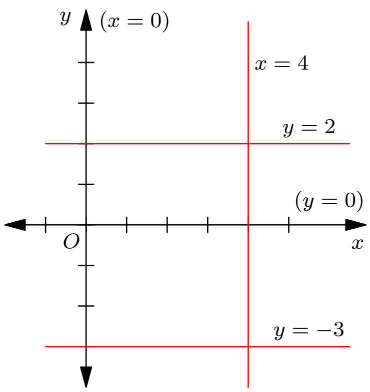
When sketching the **graph of an equation** such as $y = 2x + 1$ (i.e., joining plotted points of the graph), care must be taken to secure reasonable accuracy. In this case of a straight line, you need a sharp pencil and a ruler with a really straight edge to obtain maximum precision. Sketching curves, as in Topic 8, requires only a genuine approximation to the shape. In any case, your drawing should not look as if it had been done by Clancy of the Overflow's shearing mate "with a thumbnail dipped in tar".

6.2 Lines parallel to the axes: y=constant, x=constant

Lines $y =$ constant are parallel to the x-axis $y = 0$.

Lines $x =$ constant are parallel to the y-axis $x = 0$.

Examples in the graph: $y = 2$, $x = 4$, $y = -3$.



[The symbol || means "is (are) parallel to".]

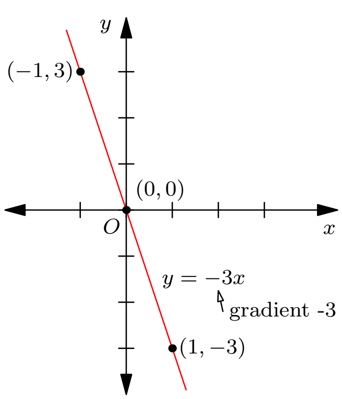
6.3 Gradient (Slope)

gradient (slope)

$y = mx \quad (= -3x \text{ say})$

$x =$	-1	0	1
$y =$	3	0	-3

(Any) *two* points determine a line uniquely, **but** a third point is useful for checking.



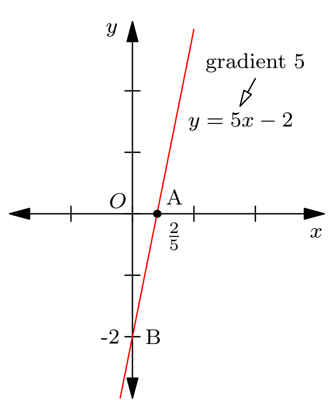
6.4 $y = mx + c$

Example: $y = 5x - 2$

$x =$	$\frac{2}{5}$	0	1
$y =$	0	-2	3

The *x-intercept* is the distance OA $(= \frac{2}{5})$. To get it, we put $y = 0$ in the equation of the line.

The *y-intercept* is the distance OB $(= -2)$. To get it, we put $x = 0$ in the equation of the line.

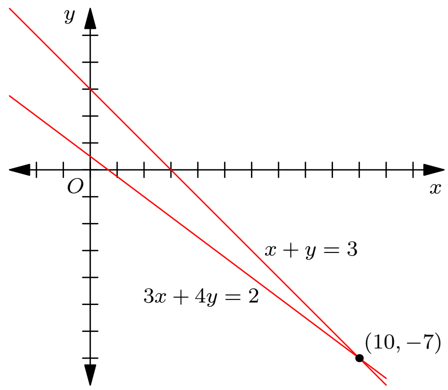


6.5 Pair of Lines

Example: Find the *unique* point of intersection of the lines

$$\left\{ \begin{array}{ccccc} 3x & + & 4y & = & 2 \\ x & + & y & = & 3 \end{array} \right. \quad \text{i.e.} \quad \left\{ \begin{array}{cccccc} y & = & - & \frac{3}{4}x & + & \frac{1}{2} \\ y & = & - & x & + & 3 \end{array} \right.$$

This problem was solved algebraically in Topic 5, Section 3 ($x = 10, y = -7$). Here, you are asked to plot points in order to draw the lines represented by the equations. Of course, the coordinates of the point of intersection of the lines must be $(10, -7)$ for total accuracy.



6.6 Parallel Lines

Two *non-parallel* lines meet in a *unique* point.

Two *parallel* lines do **not** meet. Parallel lines have the same gradient e.g. $y = 2x$ and $y = 2x + 1$ are parallel with gradient 2.

[It may be mentioned in passing that in 3 dimensions (i.e. in ordinary space) two non-parallel lines may not intersect; such lines are said to be *skew*.]

6.7 Graphing Exercises

Exercises 6.2:

Solve graphically, i.e., by drawing graphs of:

(i) $$\left.\begin{array}{rcrcr} 2x & + & 3y & = & 10 \\ 3x & - & 2y & = & -11 \end{array}\right\}$$

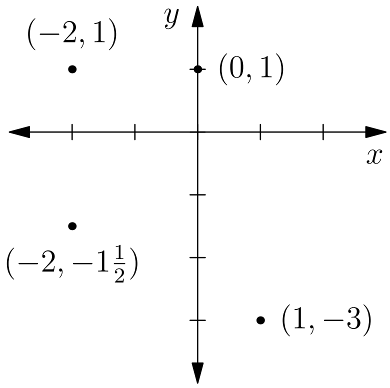
(ii) $$\left.\begin{array}{rcrcr} 5x & - & 2y & = & 1 \\ 2x & = & 4 & - & y \end{array}\right\}$$

(iii) $$\left.\begin{array}{rcrcr} x & - & y & = & 5 \\ 3x & - & 3y & = & 4 \end{array}\right\}$$

(iv) $$\left.\begin{array}{rcrcr} x & - & y & = & 5 \\ 3x & - & 3y & = & 15 \end{array}\right\}$$

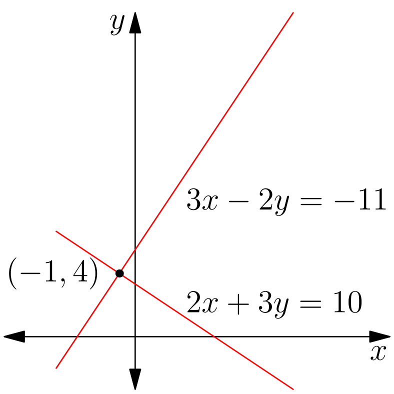
6.8 Answers to Exercises

6.1:

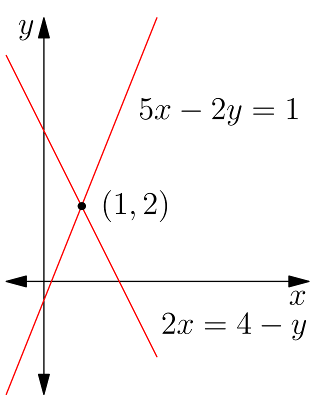


6.2:

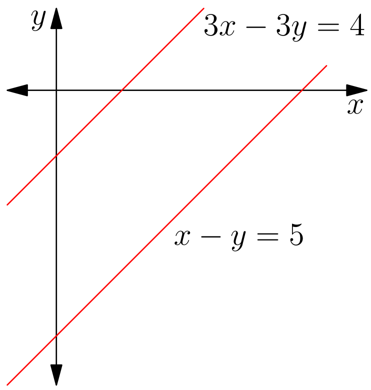
(i) $(-1, 4)$



(ii) $(1, 2)$



(iii) The lines are parallel (having the same gradient 1) so they don't met, i.e., they don't have a point of intersection, i.e., there is no solution of the equations.



(iv) The two line are not distinct, i.e., there is only **one** line, so there is an infinite number of points on the line(s), i.e., there is an infinite number of solutions.