

2 Elementary Algebra

2.1 Introduction

An algebraic symbol, i.e., a letter of an alphabet, is nothing to worry about.

A letter merely stands for a number, as we have seen in Topic 1.

Elementary Algebra is thus just a generalization of Elementary Arithmetic.

We can use any letters (or symbols) we like. There is nothing special about using the letter x , though it is the most commonly-used letter.

Question: What is meant by, say, $3x$?

Answer: $3x = 3 \times x = x + x + x = 3$ lots of x

If $x = 5$, the **value** of $3x$ is $3 \times 5 = 15$

If $x = -2$, the **value** of $3x$ is $3 \times (-2) = -6$

Notice that $3x$ also means $x \times 3 = x3$ (since the order of multiplication doesn't matter, as explained in Topic 1, Section 3). However, we usually write the number (called the *coefficient*) before the letter.

2.2 Addition and Subtraction

Examples:

$$3x + 4x = 7x \quad = \quad 4x + 3x$$

$$4x - 3x = 1x \quad = \quad x$$

$$3x - 3x = 0x \quad = \quad 0$$

$$3y + 4y = 7y \quad = \quad 4y + 3y$$

$$-5x + 7x - 6x = -4x$$

$1x = x$
$0x = 0$

Note that we can only add and subtract to make the expression simpler when the letters are the same, e.g. $3x + 4x = 7x$ and $2y + 3y = 5y$, but we can't make, say, $3x + 4y$ any simpler algebraically since x and y are different.

Exercises 2.1:

(i) $5x - 3x$

(ii) $4x - 7x$

(iii) $3y - 5y$

(iv) $8t - 7t$

(v) $3x - 7x + x$

(vi) $3x - 7y + 2x$

(vii) $4 - 3x + 2x$

(viii) $3a + 5b - 3a$

(ix) $-4x + 3x - 8x + 9x$

(x) $3x - 5w + 4w - 2x - 7 + 3w + 5 - 2w$

2.3 Multiplication

Explanation

(i) $x^0 = 1$, $x^1 = x$, $x^2 = \underbrace{x \times x}_{2 \text{ factors}}$, $x^3 = \underbrace{x \times x \times x}_{3 \text{ factors}}$, $x^4 = \underbrace{x \times x \times x \times x}_{4 \text{ factors}}$

(ii) $3x^2$ means $3 \times x \times x$ **but** $(3x)^2 = (3x) \times (3x) = 3 \times 3 \times x \times x = 9x^2$

(iii) $5xy$ means $5 \times x \times y$ ($= 5yx$ also)

(iv) $7x \times 4y = 7 \times x \times 4 \times y = 28xy$

(v) $2ab^2 = 2 \times a \times b \times b$ **but** $(2ab)^2 = 2ab \times 2ab = 2 \times a \times b \times 2 \times a \times b = 4a^2b^2$

(vi) $3x^0 = 3 \times x^0 = 3 \times 1 = 3$ **but** $(3x)^0 = 1$

Examples:

$$\begin{aligned}2(3x) &= 6x [= 3x + 3x] \\3(-2x) &= -6x \\-2(-4x) &= 8x \\-x(-x) &= (-x)^2 = x^2 \\3x(-2x) &= -6x^2 \\(-2y) \times 3x &= -6xy\end{aligned}$$
$$\left. \begin{array}{l}3x + 5xy \\4x - 3 \\2ab + 3ab^2\end{array} \right\} \text{ can't be made any simpler}$$

Exercises 2.2: Simplify (= make simpler, if possible)

- (i) $3 \times t \times z$
- (ii) $a \times 4$
- (iii) $x \times y \times x \times x$
- (iv) $2x(-x)$
- (v) $5xt - xt$
- (vi) $-4(2x) + (-3x)3$
- (vii) $3xy + 5 - 2yx$
- (viii) $(-3x)^2$
- (ix) $4x^2y - 3xy - 7yx^2 + 4$
- (x) $5x^0$
- (xi) Explain why $2xy^2 \neq 2x^2y^2 \neq (2xy)^2$

2.4 Brackets

With brackets proceed here as you did in Elementary Arithmetic, Topic 1, Section 5.

Examples:

$$3(x - 1) = 3 \times x - 3 \times 1 = 3x - 3$$

$$-(x - 2) = -x + 2 \quad \text{Note that the left hand side implies } -1 \times (x - 2)$$

$$3x - 4 - 2(x - 2) = 3x - 4 - 2x + 4 = 1x + 0 = x$$

$$x^2 + x + 1 - (x^2 - x + 1) = x^2 + x + 1 - x^2 + x - 1 = 2x$$

Exercises 2.3:

(i) $2(x + 1)$

(ii) $-(3 - x^2)$

(iii) $-2(x - 3)$

(iv) $4(x - 1) - 3(2x + 5)$

(v) $3(2 - 5x) + 4(3x + 4)$

(vi) $2(3x - 6) - 3(2x - 4)$

2.5 Division

Examples:

$$\boxed{\frac{x}{x} = 1 \text{ if } x \neq 0} \quad \frac{6x}{3x} = 2 \quad \frac{-2x}{x} = -2 \quad \frac{-5x}{-x} = 5$$

$$\frac{8y}{2} = 4y \quad \frac{4xy}{2yx} = 2 \quad \frac{6x^2}{x^2} = 6$$

Negative indices

$$\boxed{x^{-1} = \frac{1}{x}}$$

$$\boxed{x^{-2} = \frac{1}{x^2}}$$

(See also Topic 1, Section 6)

Examples:

$$(2x)^{-1} = \frac{1}{2x}$$

$$(2x)^{-2} = \frac{1}{(2x) \times (2x)} = \frac{1}{4x^2}$$

$$2x^{-2} = 2 \times x^{-2} = 2 \times \frac{1}{x \times x} = \frac{2}{x^2}$$

$$6x(3x)^{-1} = 6x \times \frac{1}{3x} = \frac{6x}{3x} = 2$$

$$(-3x)^{-1}(2x)^{-1} = \frac{1}{-3x} \times \frac{1}{2x} = \frac{1}{-6x^2} = \frac{-1}{6x^2} = -\frac{1}{6x^2}$$

Exercises 2.4:

(i) $\frac{-8x}{2x}$

(ii) $\frac{-15t}{-3t}$

(iii) $\frac{8y}{4}$

(iv) $10x(-5x)^{-1}$

(v) $(-2x)^{-1}4x$

Topic 2 Revision Exercises 2.5:

(i) $-7x + 4x - 4x$

(ii) $2x^2 + 3x^2$

(iii) $3x + 6 - 4x$

(iv) $4x^2 - 2x(-3x) - 5x(2x)$

(v) $5(x - 2) - 4(x - 3)$

(vi) $\frac{-12}{4x}$

(vii) $-20x(-4x)^{-1}$

(viii) $-6y(2y)^{-1} + (-4y)^{-1}(-8y)$

2.6 Answers to Exercises

2.1:

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|-------------|----------------|---------------|-------------|
| (i) $2x$ | (iv) t | (vii) $4 - x$ | (x) $x - 2$ |
| (ii) $-3x$ | (v) $-3x$ | (viii) $5b$ | |
| (iii) $-2y$ | (vi) $5x - 7y$ | (ix) 0 | |

2.2:

- | | | | |
|--------------|--------------|-------------------------|---------|
| (i) $3tz$ | (iv) $-2x^2$ | (vii) $5 + xy$ | (x) 5 |
| (ii) $4a$ | (v) $4xt$ | (viii) $9x^2$ | |
| (iii) x^3y | (vi) $-x$ | (ix) $-3x^2y - 3xy + 4$ | |
- (xi) We have $2 \times x \times y \times y \neq 2 \times x \times x \times y \times y \neq 4 \times x \times y \times x \times y$

2.3:

- | | | |
|-----------------------------|-----------------|----------------|
| (i) $2x + 2$ | (iii) $-2x + 6$ | (v) $-3x + 22$ |
| (ii) $-3 + x^2 (= x^2 - 3)$ | (iv) $-2x - 19$ | (vi) 0 |

2.4:

- | | | |
|----------|------------|----------|
| (i) -4 | (iii) $2y$ | (v) -2 |
| (ii) 5 | (iv) -2 | |

2.5:

- | | | | |
|-------------|----------------|---------------------|-------------|
| (i) $-6x$ | (iii) $-x + 6$ | (v) $x + 2$ | (vii) 5 |
| (ii) $5x^2$ | (iv) 0 | (vi) $-\frac{3}{x}$ | (viii) -1 |