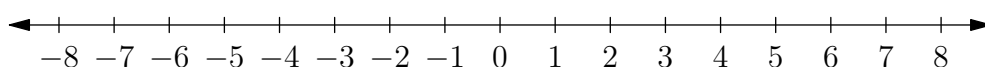


1 Elementary Arithmetic: Integers

1.1 Introduction

The integers, i.e., the positive and negative whole numbers and zero, namely, $\dots -2, -1, 0, 1, 2, \dots$, with which you are familiar may be represented on a *number line*, as drawn below.

This visual picture of positive and negative numbers (and zero) may sometimes be helpful to you in arithmetical calculations.



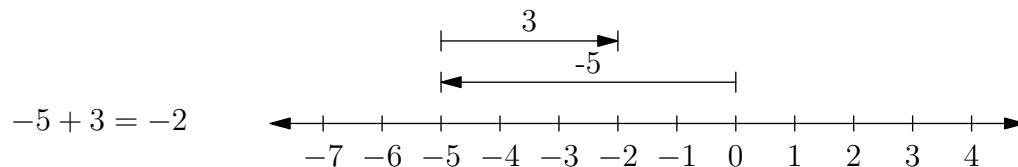
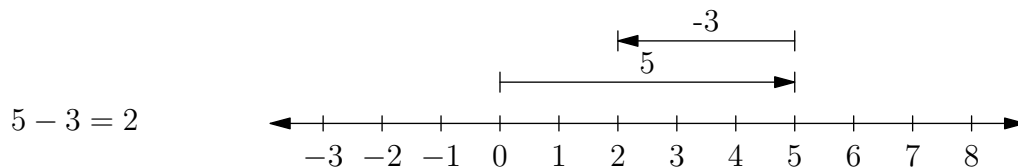
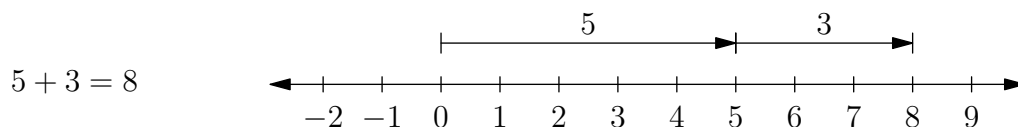
Number Line for Integers

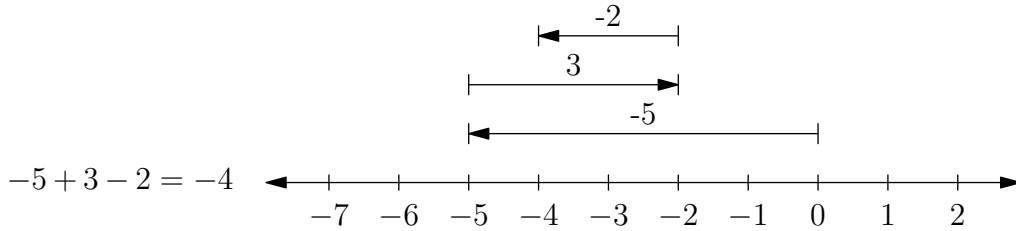
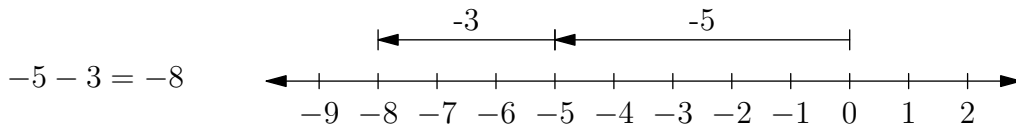
Your accuracy in calculations may be sharpened if you think of the numbers as referring to your money. (Historically, the idea of a negative number was associated with a loss in a commercial transaction, in contrast to a positive number which meant a profit.)

1.2 Addition and Subtraction

Use the number line to look at simple calculations.

Examples:





The order of addition does **not** matter, e.g. $5 + 3 = 8 = 3 + 5$.

The order of subtraction **does** matter, e.g. $5 - 3 = 2$, but $3 - 5 = -2$.

Addition and subtraction exercises 1.1:

(i) $-3 + 4$

(ii) $3 - 3$

(iii) $3 - 7$

(iv) $-3 + 8 - 2$

(v) $-3 - 4 - 2$.

Answers at the end of topic.

1.3 Multiplication

Multiplication of two positive numbers results in a positive number.

$$\boxed{+ \times + = +}$$

The order of multiplication does **not** matter,

e.g. $\underbrace{2 \times 3 = 6}_{\text{factors of 6}} = \underbrace{3 \times 2}_{\text{factors of 6}}$

Multiplication of a positive and a negative number results in a negative number:

$$\begin{array}{l} - \times + = - \\ + \times - = - \end{array}$$

i.e. Multiplication of two numbers of opposite signs gives a negative number

$$\text{e.g. } 3 \times (-2) = -6 \quad -2 \times 3 = -6 \quad 3[-(2 \times 3) = -(3 \times 2)]$$

$3 \times (-2)$ may also be written as $3(-2)$. **i.e. if there is no sign between 3 and (-2) then multiplication is understood.** We enclose -2 in **brackets** () to avoid ambiguity.

Multiplication of two negative numbers results in a positive number:

$$- \times - = +$$

Multiplication of two numbers of the same sign gives a positive number

$$\text{e.g. } -3 \times (-4) = 12 = -4 \times (-3)$$

Suppose we now agree that the letter 'a' stands for any number.

Multiplication by 0: $a \times 0 = 0 = 0 \times a$

$$\text{e.g. } 5 \times 0 = 0 = 0 \times 5 \quad 0 \times 0 = 0$$

Multiplication by 1: $a \times 1 = a = 1 \times a$

$$\begin{array}{l} \text{e.g. } -3 \times 1 = -3 = 1 \times (-3) \\ 0 \times 1 = 0 = 1 \times 0 \\ 1 \times 1 = 1 \end{array}$$

Multiplication Exercises 1.2:

(i) $2 \times (-3)$

(ii) $-4 \times (-2)$

(iii) -1×1

(iv) $0 \times (-7)$

(v) $3 \times 2 \times (-2)$

(vi) $-4 \times (-1) \times (-3)$

(vii) $-2 \times 3 \times (-2)$

1.4 Division

$$\frac{12}{4} = 3 \quad \text{means the same as} \quad 12 = 3 \times 4.$$

Division involving negative numbers obeys the **same** rules as we used in multiplication (Section 1.3), e.g.,

$$\frac{-12}{4} = -3 \quad \frac{12}{-4} = -3 \quad \frac{-12}{-4} = 3$$

Symbols such as — , \div , $/$ may all be used for division.

Note:

$$\underbrace{(12 \div 6) \div 2 = 2 \div 2 = 1} \neq \underbrace{12 \div (6 \div 2) = 12 \div 3 = 4}$$

The order of the division **is** important.

Always do the calculation in the brackets first. [\neq means ‘does **not** equal’.]

If a is any non-zero number, (i.e. $a \neq 0$), then $\frac{0}{a} = 0$

Division by 0 is IMPOSSIBLE i.e. $\frac{a}{0}$ has no meaning

Exercises 1.3:

(i) $\frac{-6}{3}$

(ii) $\frac{8}{-4}$

(iii) $\frac{-15}{-5}$

(iv) $\frac{0}{7}$

(v) $\frac{7}{0}$

(vi) $\frac{0}{-1}$

(vii) $\frac{-1}{0}$

(viii) $\frac{0}{0}$

(ix) $50 \div (10 \div 5)$

(x) $(50 \div 10) \div 5$

1.5 Use of Brackets

Examples:

$$-(-3) = 3 \quad \text{Think also of } -(-3) \text{ as } -1 \times (-3)$$

$$-5 - (-3) = -5 + 3 = -2$$

$$-2(-3) = 6$$

$$3(2 - 5) = 3(-3) = -9 \quad [\text{or } 3(2 - 5) = 6 - 15 = -9]$$

$$-4(3 - 1) = -4 \times 2 = -8$$

$$\underbrace{6 - (3 - 2)}_{= 6 - 1 = 5} \neq \underbrace{(6 - 3) - 2}_{= 3 - 2 = 1}$$

The order of brackets **is** important

i.e., do the brackets first.

Exercises 1.4

(i) $-3(6 - 8)$

(ii) $-(-3 + 5)$

(iii) $(2 - 5) - (3 - 4)$

(iv) $2 - 5 - 3 - 4$

(v) $-(4 - 3) + 2(-1 + 3)$

(vi) $4(8 - 3) - 5(3 - 7)$

(vii) $3(5 - 7) + 2(7 - 4)$

1.6 Indices

Singular *index* \equiv “pointer” (Latin).

$$\begin{array}{ccc} \text{index 0} & \text{index 1} & \text{index 2} \\ \underbrace{3^0} & \underbrace{3^1} & \underbrace{3^2} \\ = 1 & = 3 & = \underbrace{3 \times 3}_{2 \text{ factors}} = 9 \\ \\ \text{index 3} & \text{index 4} & \\ \underbrace{3^3} & \underbrace{3^4} & \\ = \underbrace{3 \times 3 \times 3}_{3 \text{ factors}} = 27 & = \underbrace{3 \times 3 \times 3 \times 3}_{4 \text{ factors}} = 81 & \end{array}$$

e.g.

$$\begin{aligned} (-3)^0 &= 1 & (-3)^1 &= -3 & (-3)^2 &= (-3) \times (-3) = 9 \\ (-3)^3 &= (-3) \times (-3) \times (-3) = -27 \end{aligned}$$

Generally,
$$a^m = \underbrace{a \times a \times a \times \cdots \times a}_{m \text{ factors}}$$

Observe that $-2^4 = -(2) \times (2) \times (2) \times (2) = -16$

while $(-2)^4 = (-2) \times (-2) \times (-2) \times (-2) = 16$

i.e., use brackets to avoid ambiguity.

$$a^0 = 1, \quad a \neq 0$$

0^0 has no meaning

Exercises 1.5:

(i) 2^5

(ii) $(-2)^3$

(iii) $(-1)^7$

(iv) $(-1)^{10}$

(v) $(-1)^0$

(vi) 437^0

Negative Indices

The negative sign in the index of a number tells us to divide 1 by that number with index made positive.

Examples:

$$3^{-1} = \frac{1}{3}, \quad 3^{-2} = \frac{1}{3^2} = \frac{1}{9}, \quad 3^{-3} = \frac{1}{3^3} = \frac{1}{27}, \quad 3^{-4} = \frac{1}{3^4} = \frac{1}{81},$$

$$(-2)^{-5} = \frac{1}{(-2)^5} = \frac{1}{(-2) \times (-2) \times (-2) \times (-2) \times (-2)} = -\frac{1}{32}$$

Generally,

$$a^{-m} = \frac{1}{a^m} = \frac{1}{\underbrace{a \times a \times a \times \cdots \times a}_{m \text{ factors}}}$$

The number a^{-m} ($= \frac{1}{a^m}$) is called the *reciprocal* of a^m .

If we multiply a number by its reciprocal we get 1,

e.g. $2 \times 2^{-1} = 2 \times \frac{1}{2} = 1$
 $3^2 \times 3^{-2} = 9 \times \frac{1}{9} = 1$

Exercises 1.6:

(i) 2^{-2}

(ii) 10^{-3}

(iii) $(-2)^{-3}$

(iv) $(-1)^{-1}$

(v) $(-5)^{-3}$

(vi) $(-3)^{-5}$

(vii) $\frac{1}{100} = 10^?$

(viii) $\frac{1}{64} = 2^?$

(ix) $\frac{1}{125} = 5^?$

1.7 Order of Operations

Previously, in Sections 2, 4, 5, you have seen the importance of the order in which operations are carried out.

The order for performing operations is:

$$\left\{ \begin{array}{l} \text{brackets} \\ \text{indices (exponents)} \\ \text{multiplication and division as it occurs from left to right} \\ \text{addition and subtraction as it occurs from left to right} \end{array} \right.$$

$$\begin{aligned} \text{e.g. } 3 + 5 \times 2 &= 3 + (5 \times 2) = 13 && (\text{not } 16 = (3 + 5) \times 2) \\ 3 + 7 \div 5 &= 3 + (7 \div 5) = 4\frac{2}{5} && (\text{not } 2 = (3 + 7) \div 5) \\ 16 \div 4 \times 2 &= (16 \div 4) \times 2 = 8 && (\text{not } 2 = 16 \div (4 \times 2)). \end{aligned}$$

To help you remember the rules about the order in which you do operations, remember the phrase PLEASE EXCUSE MY DEAR AUNT SALLY.

Do what is in **P**arentheses first

Do any **E**xponents or square roots

Do **M**ultiplication and/or **D**ivision (multiplication and division are interchangeable)

Do **A**ddition and/or **S**ubtraction (addition and subtraction are interchangeable meaning that either can go first)

Topic Revision Exercise 1.7:

(i) $-(-8 + 9) + 3^0$

(ii) $-5(-7) - 3(-6) + (-8)7$

(iii) $(-2)^3 + 3^2 - (-1)^3$

(iv) $(-3)^2 + 5 \times (-2)$

(v) $24 \div 6 \times 3$

1.8 Answers to Exercises

1.1:

(i) 1 (iii) -4 (v) -9

(ii) 0 (iv) 3

1.2:

(i) -6 (iii) -1 (v) -12 (vii) 12

(ii) 8 (iv) 0 (vi) -12

1.3:

(i) -2 (iv) 0 (vii) no meaning (x) 1

(ii) -2 (v) no meaning (viii) no meaning

(iii) 3 (vi) 0 (ix) 25

1.4:

(i) 6 (iii) -2 (v) 3 (vii) 0

(ii) -2 (iv) -10 (vi) 40

1.5:

(i) 32 (iii) -1 (v) 1

(ii) -8 (iv) 1 (vi) 1

1.6:

(i) $\frac{1}{4}$ (iv) -1 (vii) -2

(ii) $\frac{1}{1000}$ (v) $-\frac{1}{125}$ (viii) -6

(iii) $-\frac{1}{8}$ (vi) $-\frac{1}{243}$ (ix) -3

1.7:

(i) 0 (iii) 2 (v) 12

(ii) -3 (iv) -1